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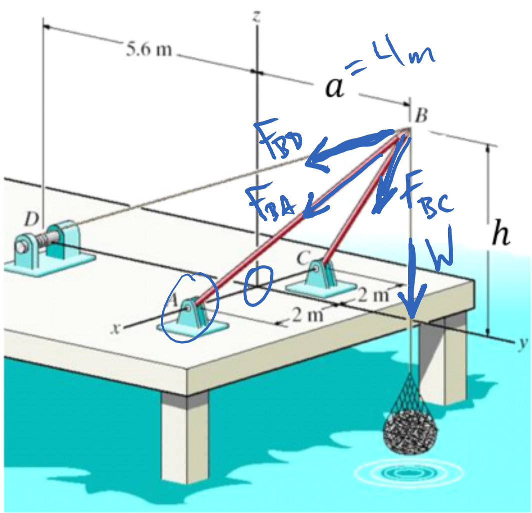
This shear leg derrick is to be designed to lift a maximum of M=200 kg of fish. Find the magnitude of the forces acting in the cable and derrick legs. Use $a = h = 4$ m. What happens to these forces when the offset distance decreases, i.e., during the lifting of the fish net until the legs are at a perpendicular position?

as $a \to 0$



Position vectors

$\underset{\sim}{r}_B = \langle 0, a, h \rangle \qquad \underset{\sim}{r}_A = \langle 2m, 0, 0 \rangle$

$\underset{\sim}{r}_C = \langle -2m, 0, 0 \rangle \qquad \underset{\sim}{r}_D = \langle 0, -5.6m, 0 \rangle$

$\underset{\sim}{r}_{BA} = \underset{\sim}{r}_A - \underset{\sim}{r}_B = \langle 2m, -a, -h \rangle$

$\underset{\sim}{r}_{BD} = \underset{\sim}{r}_D - \underset{\sim}{r}_B = \langle 0, (-5.6m - a), -h \rangle$

$\underset{\sim}{r}_{BC} = \underset{\sim}{r}_C - \underset{\sim}{r}_B = \langle -2m, -a, -h \rangle$

unit vectors

$$\hat{u}_{BA} = \frac{\underset{\sim}{r}_{BA}}{|\underset{\sim}{r}_{BA}|} = \frac{\langle 2m, -a, -h \rangle}{\sqrt{(2m)^2 + a^2 + h^2}}$$

$$\hat{u}_{BD} = \frac{\langle 0, (-5.6m - a), -h \rangle}{\sqrt{(5.6m + a)^2 + h^2}}$$

$$\hat{u}_{BC} = \frac{\langle -2m, -a, -h \rangle}{\sqrt{(2m)^2 + a^2 + h^2}}$$

unknown

Force vectors:

$$\underset{\sim}{F}_{BA} = F_{BA} \cdot \hat{u}_{BA} = \frac{F_{BA} \langle 2m, -a, -h \rangle}{\sqrt{(2m)^2 + a^2 + h^2}}$$

$$\underset{\sim}{F}_{BC} = \frac{F_{BC} \langle -2m, -a, -h \rangle}{\sqrt{(2m)^2 + a^2 + h^2}}$$

$$\underset{\sim}{F}_{BD} = \frac{F_{BD} \langle 0, (-5.6m - a), -h \rangle}{\sqrt{(5.6m + a)^2 + h^2}}$$

3 knowns

3 ~~unknowns~~ $\sqrt{(5.6m+a)^2+h^2}$

3 equations: $\underset{\sim}{F}_{BA} + \underset{\sim}{F}_{BC} + \underset{\sim}{F}_{BD} + \underset{\sim}{W} = m \cdot \cancel{\underset{\sim}{a}_B} = \underset{\sim}{0}$

$$F_{BA} \cdot \frac{\langle 2m, -a, -h \rangle}{\sqrt{(2m)^2 + a^2 + h^2}} + F_{BC} \cdot \frac{\langle -2m, -a, -h \rangle}{\sqrt{(2m)^2 + a^2 + h^2}} + \frac{F_{BD} \cdot \langle 0, (-5.6m - a), -h \rangle}{\sqrt{h^2 + (5.6m+a)^2}} + m \cdot g \langle 0, 0, -1 \rangle = \langle 0, 0, 0 \rangle$$

In $\hat{i}$: $$\frac{F_{BA}(2m) + F_{BC} \cdot (-2m)}{\sqrt{(2m)^2 + a^2 + h^2}} + 0 + 0 = 0$$

$$\Rightarrow F_{BA} = F_{BC}$$

In $\hat{j}$: $$\frac{(F_{BA} + \overset{F_{BA}}{F_{BC}})(-a)}{\sqrt{(2m)^2 + a^2 + h^2}} + \frac{F_{BD} \cdot (-5.6m - a)}{\sqrt{h^2 + (5.6m+a)^2}} + 0 = 0$$

$$\therefore F_{BA} = -F_{BD} \cdot \left(\frac{5.6m + a}{2a} \right) \sqrt{\frac{(2m)^2 + a^2 + h^2}{(5.6m+a)^2 + h^2}}$$ ☆

In $\hat{k}$: $$\frac{F_{BA} \cdot (-h) + F_{BA} \cdot (-h)}{\sqrt{(2m)^2 + a^2 + h^2}} - \frac{F_{BD} \cdot h}{\sqrt{(5.6m+a)^2 + h^2}} - m \cdot g = 0$$

Sub. eqn. ✪ into $\hat{k}$ eqn.:

$$\frac{-2\cdot h\cdot(-F_{BD})}{\sqrt{(2m)^2+a^2+h^2}}\cdot\left(\frac{5.6m+a}{2a}\right)\sqrt{\frac{(2m)^2+a^2+h^2}{(5.6m+a)^2+h^2}}-\frac{F_{BD}\cdot h}{\sqrt{(5.6m+a)^2+h^2}}-mg=0$$

$\therefore$ solve for F_{BD}:

$$\boxed{F_{BD}=m\cdot g\cdot\frac{a}{h}\cdot\frac{\sqrt{(5.6m+a)^2+h^2}}{5.6m}}$$

tension in cable

if $a\to 0$ as $a\to 0$, $F_{BD}\to 0$

In the legs (solve for $\underset{\sim}{F}_{BA}$):

From ✪

$$F_{BA}=-m\cdot g\cdot\frac{\cancel{a}}{h}\cdot\frac{\cancel{\sqrt{(5.6m+a)^2+h^2}}}{5.6m}\cdot\left(\frac{5.6m+a}{2\cancel{a}}\right)\sqrt{\frac{(2m)^2+a^2+h^2}{\cancel{(5.6m+a)^2+h^2}}}$$

$$\boxed{F_{BA}=-m\cdot g\cdot\frac{(\cancel{5.6m+a})}{2\cdot h\cdot(\cancel{5.6m})}\sqrt{(2m)^2+\cancel{a^2}+h^2}}$$

force in legs is in compression

as $a\to 0$

$$\boxed{F_{BA}=-\frac{m\cdot g}{2h}\sqrt{(2m)^2+h^2}}$$

still in compression, but lower magnitude

[Inclass_v2_Chapter4_ForceSystemResultant.pptx]

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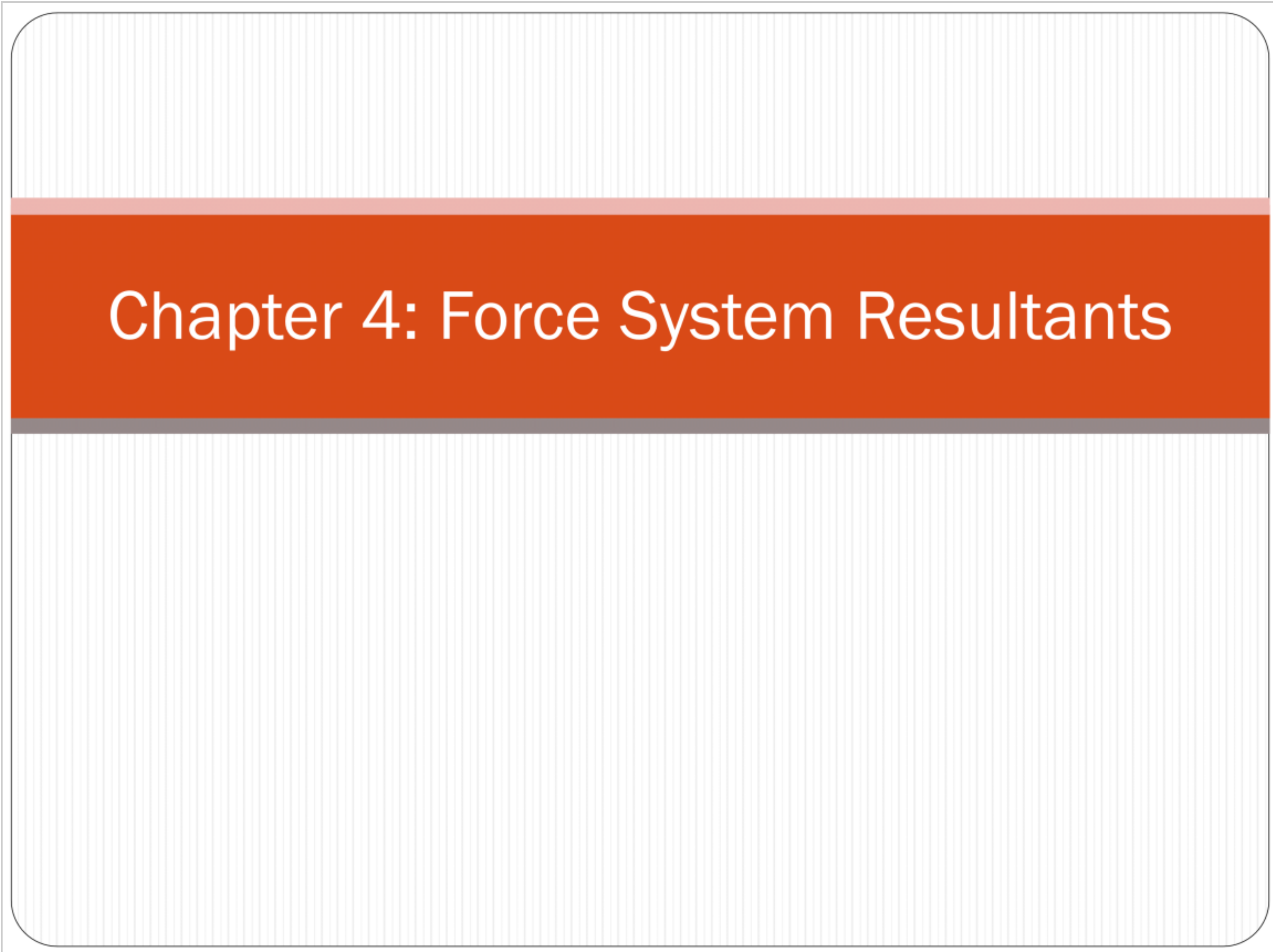

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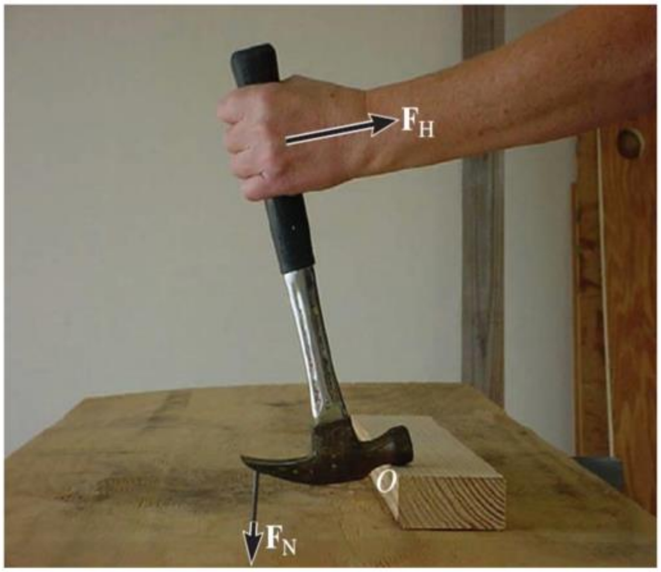
Applications



Carpenters often use a hammer in this way to pull a stubborn nail. Through what sort of action does the force F_H at the handle pull the nail? How can you mathematically model the effect of force F_H at point O?

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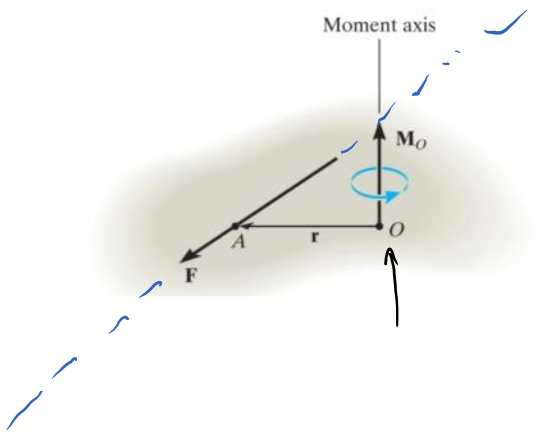
Moment of a force – vector formulation

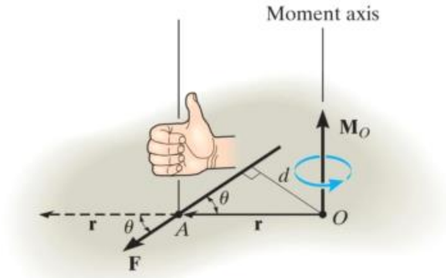
The **moment of a force about a point** provides a measure of the **tendency for rotation** (sometimes called a torque).

The moment of a force **F** about point **O**, or actually about the moment axis passing through O and perpendicular to the plane containing **O** and **F**, can be expressed using the cross (vector) product, namely:

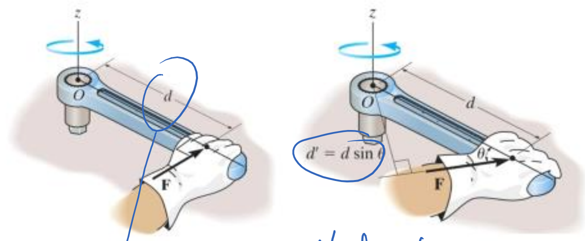
$$\boldsymbol{M}_O = \boldsymbol{r} \times \boldsymbol{F}$$

where $\underset{\sim}{r}$ is the position vector directed from **O** to any point on the line of action of **F**.

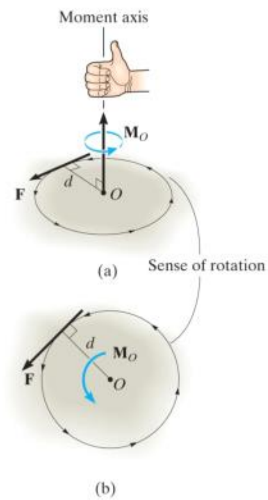




Moment of a force – scalar formulation



$d' = d \cdot \sin\theta$



Magnitude: In a 2-D case, the magnitude of the moment is $M_o = F\,d$

Direction: The moment is perpendicular to the plane that contains the force **F** and its moment arm d. The right-hand rule is used to define the sense.

Scalar form: $|\underset{\sim}{M}_o| = |\underset{\sim}{r}| \cdot |\underset{\sim}{F}| \cdot \sin\theta$

$$\underset{\sim}{C} = \underset{\sim}{A} \times \underset{\sim}{B}$$
$$|\underset{\sim}{C}| = |\underset{\sim}{A}| \cdot |\underset{\sim}{B}| \cdot \sin\theta_{AB}$$

$= r \cdot F \cdot \sin\theta$

$= F \cdot d$ (here $d = r \cdot \sin\theta$)